

Distributed Detection/Isolation Procedures for Quickest Event Detection in Large Extent Wireless Sensor Networks

K. Premkumar[§], Anurag Kumar[†], and Joy Kuri[‡]

Abstract

We study a problem of distributed detection of a stationary point event in a large extent wireless sensor network (WSN), where the event influences the observations of the sensors only in the vicinity of where it occurs. An event occurs at an unknown time and at a random location in the coverage region (or region of interest (ROI)) of the WSN. We consider a general sensing model in which the effect of the event at a sensor node depends on the distance between the event and the sensor node; in particular, in the Boolean sensing model, all sensors in a disk of a given radius around the event are equally affected. Following the prior work reported in [1], [2], [3], *the problem is formulated as that of detecting the event and locating it to a subregion of the ROI as early as possible under the constraints that the average run length to false alarm (ARL2FA) is bounded below by γ , and the probability of false isolation (PFI) is bounded above by α* , where γ and α are target performance requirements. In this setting, we propose distributed procedures for event detection and isolation (namely MAX, ALL, and HALL), based on the local fusion of CUSUMs at the sensors. For these procedures, we obtain bounds on the maximum mean

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detection/isolation delay (SADD), and on ARL2FA and PFI, and thus provide an upper bound on SADD as $\min\{\gamma, 1/\alpha\} \rightarrow \infty$. For the Boolean sensing model, we show that an asymptotic upper bound on the maximum mean detection/isolation delay of our distributed procedure scales with γ and α in the same way as the asymptotically optimal centralised procedure [2].

Index Terms

Disorder problem, distributed quickest change detection, detection with distance dependent sensing, fusion of CUSUMs, multi-decision change-point detection, multi-hypothesis change detection

I. INTRODUCTION

Event detection is an important application for which a wireless sensor network (WSN) is deployed. A number of sensor nodes (or “motes”) that can sense, compute, and communicate are deployed in a region of interest (ROI) in which the occurrence of an event (e.g., crack in a structure) has to be detected. In our work, we view *an event as being associated with a change in the distribution (or cumulative distribution function) of a physical quantity that is sensed by the sensor nodes*. Thus, the work we present in this paper is in the framework of quickest detection of change in a random process. In the case of small extent networks, where the coverage of every sensor spans the whole ROI, and where we assume that an event affects all the sensor nodes in a statistically equivalent manner, we obtain the classical change detection problem whose solution is well known (see, for example, [4], [5], [6]). In [7] and [8], we have studied variations of the classical problem in the WSN context, where there is a wireless communication network between the sensors and the fusion centre [7], and where there is a cost for taking sensor measurements [8].

However, in the case of large extent networks, where the ROI is large compared to the coverage region of a sensor, an event (e.g., a crack in a huge structure, gas leakage from a joint in a storage tank) affects sensors that are in its proximity; further the effect depends on the distances of the sensor nodes from the event. Since the location of the event is unknown, *the post-change distribution of the observations of the sensor nodes are not known*. In this paper, we are interested in obtaining procedures for detecting and locating an event in a large extent network. This problem is also referred to as *change detection and isolation* (see [1], [2], [3], [9], [10]). Since the ROI is large, a large number of sensors are deployed to cover the ROI, making a centralised solution infeasible. In our work, *we seek distributed algorithms for detecting and locating an event, with small detection delay, subject to constraints on false alarm and false isolation*. The distributed algorithms require only local information from the neighborhood of each node.

A. Discussion of Related Literature

The problem of sequential change detection/isolation with a finite set of post-change hypotheses was introduced by Nikiforov [1], where he studied the change detection/isolation problem with the observations being conditionally independent, and proposed a non-Bayesian procedure which is shown to be maximum mean detection/isolation delay optimal, as the average run lengths to false alarm and false isolation go to ∞ . Lai [10] considered the multi-hypothesis change detection/isolation problem with stationary pre-change and post-change observations, and obtained asymptotic lower bounds for the maximum mean detection/isolation delay.

Nikiforov also studied a change detection/isolation problem under the average run length to false alarm (ARL2FA) and the probability of false isolation (PFI) constraints [2], in which he showed that a CUSUM-like *recursive* procedure is asymptotically maximum mean detection/isolation delay optimal among the procedures that satisfy $\text{ARL2FA} \geq \gamma$ and $\text{PFI} \leq \alpha$ asymptotically, as $\min\{\gamma, 1/\alpha\} \rightarrow \infty$. Tartakovsky in [3] also studied the change detection/isolation problem where he proposed recursive matrix CUSUM and recursive matrix Shirayev–Roberts tests, and showed that they are asymptotically maximum mean delay optimal over the constraints $\text{ARL2FA} \geq \gamma$ and $\text{PFI} \leq \alpha$ asymptotically, as $\min\{\gamma, 1/\alpha\} \rightarrow \infty$.

Malladi and Speyer [11] studied a Bayesian change detection/isolation problem and obtained a mean delay optimal centralised procedure which is a threshold based rule on the a posteriori probability of change corresponding to each post-change hypothesis.

Centralised procedures incur high communication costs and distributed procedures would be desirable. In this paper, we study distributed procedures based on CUSUM detectors at the sensor nodes where the CUSUM detector at sensor node s is driven only by the observations made at node s . Also, in the case of large extent networks, the post-change distribution of the observations of a sensor node, in general, depends on the distance between the event and the sensor node which is unknown.

B. Summary of Contributions

- 1) As the WSN considered is of large extent, the post-change distribution is unknown, and could belong to a set of alternate hypotheses. In Section III, we formulate the event detection/isolation problem in a large extent network in the framework of [2], [3] as a maximum mean detection/isolation delay minimisation problem subject to an average run length to false alarm (ARL2FA) and probability of false isolation (PFI) constraints.
- 2) We propose distributed detection/isolation procedures MAX, ALL, and HALL (**H**ysteresis modified **A**LL) for large extent networks in Section IV. The procedures MAX and ALL are extensions of

the decentralised procedures MAX [6] and ALL [9], [12], which were developed for small extent networks. The distributed procedures are energy-efficient compared to the centralised procedures. Also, the known centralised procedures are applicable only for the Boolean sensing model.

- 3) In Section IV, we first obtain bounds on ARL2FA, PFI, and maximum mean detection/isolation delay (SADD) for the distributed procedures MAX, ALL, and HALL. These bounds are then applied to get an upper bound on the SADD for the procedures when $\text{ARL2FA} \geq \gamma$, and $\text{PFI} \leq \alpha$, where γ and α are some performance requirements. For the case of the Boolean sensing model, we compare the SADD of the distributed procedures with that of Nikiforov's procedure [2] (a centralised asymptotically optimal procedure) and show that the an asymptotic upper bound on the maximum mean detection/isolation delay of our distributed procedure scales with γ and α in the same way as that of [2].

II. SYSTEM MODEL

Let $\mathcal{A} \subset \mathbb{R}^2$ be the region of interest (ROI) in which n sensor nodes are deployed. All nodes are equipped with the same type of sensor (e.g., acoustic). Let $\ell^{(s)} \in \mathcal{A}$ be the location of sensor node s , and define $\ell := [\ell^{(1)}, \ell^{(2)}, \dots, \ell^{(n)}]$. We consider a discrete-time system, with the basic unit of time being one slot, indexed by $k = 0, 1, 2, \dots$, the slot k being the time interval $[k, k + 1)$. The sensor nodes are assumed to be time-synchronised (see, for example, [13]), and at the beginning of every slot $k \geq 1$, each sensor node s samples its environment and obtains the observation $X_k^{(s)} \in \mathbb{R}$.

A. Change/Event Model

An event (or change) occurs at an unknown time $T \in \{1, 2, \dots\}$ and at an unknown location $\ell_e \in \mathcal{A}$. We consider only stationary (and permanent or persistent) point events, i.e., an event occurs at a point in the region of interest, and *having occurred, stays there forever*. Examples that would motivate such a model are 1) gas leakage in the wall of a large storage tank, 2) excessive strain at a point in a large 2-dimensional structure. In [14] and [15], the authors study change detection problems in which the event stays only for a finite random amount of time.

An event is viewed as a source of some physical signal that can be sensed by the sensor nodes. Let h_e be the signal strength of the event¹. A sensor at a distance d from the event senses a signal $h_e \rho(d) + W$,

¹In case, the signal strength of the event is not known, but is known to lie in an interval $[\underline{h}, \bar{h}]$, we work with $h_e = \underline{h}$ as this corresponds to the least Kullback–Leibler divergence between the “event not occurred” hypothesis and the “event occurred” hypothesis. See [16] for change detection with unknown parameters for a collocated network.

where W is a random zero mean noise, and $\rho(d)$ is the distance dependent loss in signal strength which is a decreasing function of the distance d , with $\rho(0) = 1$. We assume an isotropic distance dependent loss model, whereby the signal received by all sensors at a distance d (from the event) is the same.

Example 1 The Boolean model (see [17]): In this model, the signal strength that a sensor receives is the same (which is given by h_e) when the event occurs within a distance of r_d from the sensor and is 0 otherwise. Thus, for a Boolean sensing model,

$$\rho(d) = \begin{cases} 1, & \text{if } d \leq r_d \\ 0, & \text{otherwise.} \end{cases}$$

Example 2 The power law path-loss model (see [17]) is given by

$$\rho(d) = d^{-\eta},$$

for some path loss exponent $\eta > 0$. For free space, $\eta = 2$.

B. Detection Region and Detection Partition

In Example 2, we see that the signal from an event varies continuously over the region. Hence, unlike the Boolean model, there is no clear demarcation between the sensors that observe the event and those that do not. Thus, in order to facilitate the design of a distributed detection scheme with some performance guarantees, in the remainder of this section, we will define certain regions around each sensor.

Definition 1 Given $0 < \mu_1 \leq h_e$, the **Detection Range** r_d of a sensor is defined as the distance from the sensor within which the occurrence of an event induces a signal level of at least μ_1 , i.e.,

$$r_d := \sup \{d : h_e \rho(d) \geq \mu_1\}.$$

■

In the above definition, μ_1 is a design parameter that defines the acceptable detection delay. For a given signal strength h_e , a large value of μ_1 results in a small detection range r_d (as $\rho(d)$ is non-increasing in d). We will see in Section IV-F (Eqn. (17)) that the SADD of the distributed change detection/isolation procedures we propose, depends on the detection range r_d , and that a small r_d (i.e., a large μ_1) results in a small SADD, while requiring more sensors to be deployed in order to achieve coverage of the ROI.

We say that a location $x \in \text{ROI}$ is *detection-covered* by sensor node s , if $\|\ell^{(s)} - x\| \leq r_d$. For any sensor node s , $\mathcal{D}^{(s)} := \{x \in \mathcal{A} : \|\ell^{(s)} - x\| \leq r_d\}$ is called its *detection-coverage region* (see Fig. 1). We assume that the sensor deployment is such that every $x \in \mathcal{A}$ is *detection-covered* by at least one sensor

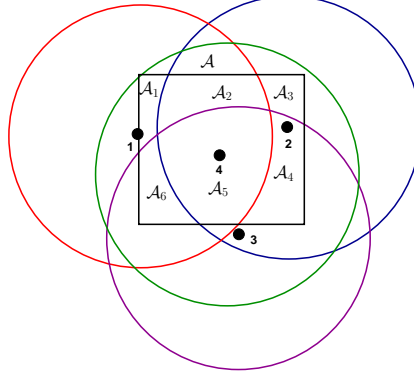


Fig. 1. **Partitioning of \mathcal{A} in a large WSN by detection regions:** (a simple example) The coloured solid circles around each sensor node denote their detection regions. The four sensor nodes divide the ROI, indicated by the square region, into regions $\mathcal{A}_1, \dots, \mathcal{A}_6$ such that region \mathcal{A}_i is detection-covered by a unique set of sensors \mathcal{N}_i . For example, \mathcal{A}_1 is detection covered by the set of sensors $\mathcal{N}_1 = \{1, 2, 4\}$, etc.

(Fig. 1). For each $x \in \mathcal{A}$, define $\mathcal{N}(x)$ to be the largest set of sensors by which x is detection-covered, i.e., $\mathcal{N}(x) := \{s : x \in \mathcal{D}^{(s)}\}$. Let $\mathcal{C}(\mathcal{N}) = \{\mathcal{N}(x) : x \in \mathcal{A}\}$. $\mathcal{C}(\mathcal{N})$ is a finite set and can have at most $2^n - 1$ elements. Let $N = |\mathcal{C}(\mathcal{N})|$. For each $\mathcal{N}_i \in \mathcal{C}(\mathcal{N})$, we denote the corresponding detection-covered region by $\mathcal{A}_i = \mathcal{A}(\mathcal{N}_i) := \{x \in \text{ROI} : \mathcal{N}(x) = \mathcal{N}_i\}$. Evidently, the $\mathcal{A}_i, 1 \leq i \leq N$, partition the ROI. We say that the ROI is *detection-partitioned* into a *minimum number of subregions*, $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$, such that the subregion \mathcal{A}_i is detection-covered by a unique set of sensors \mathcal{N}_i , and \mathcal{A}_i is the maximal detection-covered region of \mathcal{N}_i , i.e., $\forall i \neq i', \mathcal{N}_i \neq \mathcal{N}_{i'}$ and $\mathcal{A}_i \cap \mathcal{A}_{i'} = \emptyset$. See Fig. 1 for an example.

C. Sensor Measurement Model

Before change, i.e., for $k < T$, the observation $X_k^{(s)}$ at the sensor s is just the zero mean sensor noise $W_k^{(s)}$, the probability density function (pdf) of which is denoted by $f_0(\cdot)$ (*pre-change pdf*). After change, i.e., for $k \geq T$ with the location of the event being ℓ_e , the observation of sensor s is given by $X_k^{(s)} = h_e \rho(d_{e,s}) + W_k^{(s)}$ where $d_{e,s} := \|\ell^{(s)} - \ell_e\|$, the pdf of which is denoted by $f_1(\cdot; d_{e,s})$ (*post-change pdf*). The noise processes $\{W_k^{(s)}\}$ are independent and identically distributed (iid) across time and across sensor nodes. In the rest of the paper, we consider $f_0(\cdot)$ to be Gaussian with mean 0 and variance σ^2 .

We denote the probability measure when the change happens at time T and at location ℓ_e by $P_T^{(\mathbf{d}(\ell_e))} \{\cdot\}$, where $\mathbf{d}(\ell_e) = [d_{e,1}, d_{e,2}, \dots, d_{e,n}]$, and the corresponding expectation operator by $E_T^{(\mathbf{d}(\ell_e))} [\cdot]$. In the case of Boolean sensing model, the post-change pdfs depend only on the detection subregion where the event occurs, and hence, we denote the probability measure when the event occurs at $\ell_e \in \mathcal{A}_i$ and at time T

by $P_T^{(i)}\{\cdot\}$, and the corresponding expectation operator by $E_T^{(i)}[\cdot]$.

D. Local Change Detectors

We compute a CUSUM statistic $C_k^{(s)}, k \geq 1$ at each sensor s based only on its own observations. The CUSUM procedure was proposed by Page [5] as a solution to the classical change detection problem (CDP, in which there is one pre-change hypothesis and only one post-change hypothesis). The optimality of CUSUM was shown for conditionally iid observations by Moustakides in [18] for a maximum mean delay metric introduced by Pollak [19] which is $SADD(\tau) := \sup_{T \geq 1} E_T[\tau - T | \tau \geq T]$.

The driving term of CUSUM should be the log likelihood-ratio (LLR) of $X_k^{(s)}$ defined as $Z_k^{(s)}(d_{e,s}) := \ln \left(\frac{f_1(X_k^{(s)}; d_{e,s})}{f_0(X_k^{(s)})} \right)$. As the location of the event ℓ_e is unknown, the distance $d_{e,s}$ is also unknown. Hence, one cannot work with the pdfs $f_1(\cdot; d_{e,s})$. We propose to drive the CUSUM at each node s with $Z_k^{(s)}(r_d)$, where we recall that r_d is the detection range of a sensor. Based on the CUSUM statistic $C_k^{(s)}, k \geq 1$, sensor s computes a sequence of local decisions $D_k^{(s)} \in \{0, 1\}, k \geq 1$, where 0 represents no-change and 1 represents change. For each set of sensor nodes \mathcal{N}_i that detection partitions the ROI, we define $\tau^{(\mathcal{N}_i)}$, the stopping time (based on the sequence of local decisions $D_k^{(s)}$ s for all $s \in \mathcal{N}_i$) at which the set of sensors \mathcal{N}_i detects the event. The way we obtain the local decisions $D_k^{(s)}$ from the CUSUM statistic $C_k^{(s)}, k \geq 1$, and the way these local decisions determine the stopping times $\tau^{(\mathcal{N}_i)}$, varies from rule to rule. Specific rules for local decision and the fusion of local decisions will be described in Section IV (also see [20]).

An implementation strategy for our distributed event detection/isolation procedure can be the following. We assume that the sensors know to which detection sensor sets \mathcal{N}_i s they belong. This could be done by initial configuration or by self-organisation. When the local decision of sensor s is 1, it broadcasts this fact to all sensors in its detection neighbourhood. In practise, the broadcast range of these radios is substantially larger than the detection range. Hence, the local decision of s is learnt by all sensors s' that belong to \mathcal{N}_i to which s belongs. When any node learns that all the sensors in \mathcal{N}_i have reached the local decision 1, it transmits an alarm message to the base station [21]. A distributed leader election algorithm can be implemented so that only one, or a controlled number of alarms is sent. This alarm message is carried by geographical forwarding [22]. A system that utilises such local fusion (but with a different sensing and detection model) was developed by us and is reported in [23].

E. Influence Region

After a set of nodes \mathcal{N}_i declares an event, the event is *isolated* to a region associated with \mathcal{N}_i called the influence region. In the Boolean sensing model, if an event occurs in \mathcal{A}_i , then only the sensors $s \in \mathcal{N}_i$ observe the event, while the other sensors $s' \notin \mathcal{N}_i$ only observe noise. On the other hand, in the power law path-loss model, sensors $s' \notin \mathcal{N}_i$ can also observe the event, and the driving term of the CUSUMs of sensors s' may be affected by the event. The mean of the driving term of CUSUM of any sensor s is given by

$$\mathbb{E}_{f_1(\cdot; d_{e,s})}[Z_k^{(s)}(r_d)] = \frac{(h_e \rho(r_d))^2}{2\sigma^2} \left(\frac{2\rho(d_{e,s})}{\rho(r_d)} - 1 \right). \quad (1)$$

Thus, the mean of the increment that drives CUSUM of node s decreases with $d_{e,s}$ and becomes negative when $2\rho(d_{e,s}) < \rho(r_d)$. In this region, we are interested in finding T_E , the expected time for the CUSUM statistic $C_k^{(s)}$ to cross the threshold c . Define $\tau^{(s)} := \inf \{k : C_k^{(s)} \geq c\}$, and hence, $T_E = \mathbb{E}_1^{(\mathbf{d}^{(\ell_e)})} [\tau^{(s)}]$.

Lemma 1 *If the distance between sensor node s and the event, $d_{e,s}$ is such that $2\rho(d_{e,s}) < \rho(r_d)$, then*

$$T_E \geq \exp(\omega_0 c)$$

where $\omega_0 = 1 - \frac{2\rho(d)}{\rho(r_d)}$.

Proof: From (Eqn. 5.2.79 pg. 177 of) [24], we can show that $\mathbb{E}_1^{(\mathbf{d}^{(\ell_e)})} [\tau^{(s)}] \geq \exp(\omega_0 c)$ where ω_0 is the solution to the equation

$$\mathbb{E}_1^{(\mathbf{d}^{(\ell_e)})} \left[e^{\omega_0 Z_k^{(i)}(r_d)} \right] = 0,$$

which is given by $\omega_0 = 1 - \frac{2\rho(d)}{\rho(r_d)}$ (see Eqn. (1)). ■

We would be interested in $T_E \geq \exp(\underline{\omega}_0 \cdot c)$ for some $0 < \underline{\omega}_0 < 1$. We now define the *influence range* of a sensor as follows.

Definition 2 Influence Range of a sensor, r_i , is defined as the distance from the sensor within which the occurrence of an event can be detected within a mean delay of $\exp(\underline{\omega}_0 c)$ where $\underline{\omega}_0$ is a parameter of interest and c is the threshold of the local CUSUM detector. Using Lemma 1, we see that $r_i = \min\{d' : 2\rho(d') \leq (1 - \underline{\omega}_0)\rho(r_d)\}$. ■

A location $x \in \mathcal{A}$ is influence covered by a sensor s if $\|\ell^{(s)} - x\| \leq r_i$, and a set of sensors \mathcal{N}_j is said to influence cover x if each sensor $s \in \mathcal{N}_j$ influence covers x .

From Lemma 1, we see that by having a large value of $\underline{\omega}_0$, i.e., $\underline{\omega}_0$ close to 1, the sensors that are beyond a distance of r_i from the event take a long time to cross the threshold. However, we see from

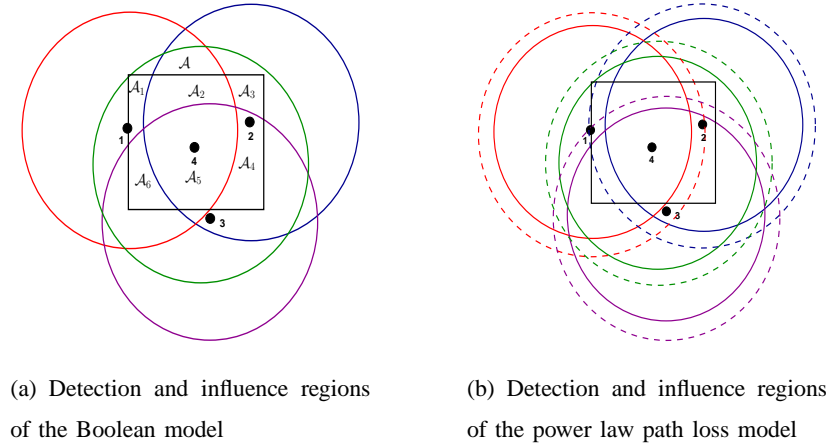


Fig. 2. **Influence and detection regions:** A simple example of partitioning of \mathcal{A} in a large WSN. The coloured solid circles around each sensor node denote their detection regions. The four sensor nodes, in the figure, divide the ROI, indicated by the square region, into regions $\mathcal{A}_1, \dots, \mathcal{A}_6$ such that region \mathcal{A}_i is detection-covered by a unique set of sensors \mathcal{N}_i . The dashed circles represent the influence regions. In the Boolean model, the influence region of a sensor coincides with its detection region.

the definition of influence range that a large value of ω_0 gives a large influence range r_i . We will see from the discussion in Section II-F that a large influence range results in the isolation of the event to a large subregion of \mathcal{A} . On the other hand, from Section IV-E, we will see that a large ω_0 decreases the probability of false isolation, a performance metric of change detection/isolation procedure, which we define in Section III.

We define the *influence-region* of sensor s as $\mathcal{T}^{(s)} := \{x \in \mathcal{A} : \|\ell^{(s)} - x\| \leq r_i\}$. For the Boolean sensing model, $r_i = r_d$, and hence, $\mathcal{D}^{(s)} = \mathcal{T}^{(s)}$ for all $1 \leq s \leq n$, and for the power law path-loss sensing model, $r_i > r_d$, and hence, $\mathcal{D}^{(s)} \subset \mathcal{T}^{(s)}$ for all $1 \leq s \leq n$ (see Fig. 2).

Recalling the sets of sensors \mathcal{N}_i , $1 \leq i \leq N$, defined in Section II-B, we define the *influence region of the set of sensors \mathcal{N}_i* as the region \mathcal{B}_i such that each $x \in \mathcal{B}_i$ is within the influence range of all the sensors in \mathcal{N}_i , i.e., $\mathcal{B}_i := \mathcal{B}(\mathcal{N}_i) := \bigcap_{s \in \mathcal{N}_i} \mathcal{T}^{(s)}$. Note that $\mathcal{A}(\mathcal{N}_i) = \left(\bigcap_{s \in \mathcal{N}_i} \mathcal{D}^{(s)} \right) \cap \left(\bigcap_{s' \notin \mathcal{N}_i} \overline{\mathcal{D}^{(s')}} \right)$, where $\overline{\mathcal{D}}$ is the complement of the set \mathcal{D} , and $\mathcal{D}^{(s)} \subseteq \mathcal{T}^{(s)}$. Hence, $\mathcal{A}(\mathcal{N}_i) \subseteq \mathcal{B}(\mathcal{N}_i)$. For the power law path-loss sensing model, $\mathcal{D}^{(s)} \subset \mathcal{T}^{(s)}$ for all $1 \leq s \leq n$, and hence, $\mathcal{A}(\mathcal{N}_i) \subset \mathcal{B}(\mathcal{N}_i)$ for all $1 \leq i \leq N$. For the Boolean sensing model, $\mathcal{A}(\mathcal{N}_i) = \mathcal{B}(\mathcal{N}_i) \cap \left(\bigcap_{s' \notin \mathcal{N}_i} \overline{\mathcal{D}^{(s')}} \right)$, and hence $\mathcal{A}(\mathcal{N}_i) = \mathcal{B}(\mathcal{N}_i)$ only when $\mathcal{N}_i = \{1, 2, \dots, n\}$. Thus, for a general sensing model, $\mathcal{A}(\mathcal{N}_i) \subseteq \mathcal{B}(\mathcal{N}_i)$. We note here that in the Boolean and the power law path loss models, an event which does not lie in the detection subregion of \mathcal{N}_i , but lies in its influence subregion (i.e., $\ell_e \in \mathcal{B}(\mathcal{N}_i) \setminus \mathcal{A}(\mathcal{N}_i)$) can be detected due to \mathcal{N}_i because of the stochastic nature of the observations; in the power law path loss sensing model, this is also because

of the difference in losses $\rho(d_{e,s})$ between different sensors.

Remark: The definition of the detection and influence ranges have involved two design parameters μ_1 and $\underline{\omega}_0$ which can be used to “tune” the performance of the distributed detection schemes that we develop. ■

F. Isolating the Event

In Section II D, we provided an outline of a class of distributed detection procedures that will yield a stopping rule. On stopping, a decision for the location of the event is made, which is called *isolation*. In Section IV, we will provide specific distributed detection/isolation procedures in which stopping will be due to one of the sensor sets \mathcal{N}_i .

An event occurring at location $\ell_e \in \mathcal{A}_i$ can influence sensors s' which influence cover ℓ_e , and hence, the detection can be due to sensors $\mathcal{N}_i \neq \mathcal{N}_j$ which influence cover ℓ_e . Thus, we isolate the event to the influence region of the sensors that detect the event. Because of noise, detection can be due to a sensor set \mathcal{N}_h which does not influence cover the event. Such an error event is called false isolation.

An event occurring at $\ell_e \in \mathcal{A}_i$ is influence covered by sensors $s' \in \mathcal{N}(\ell_e) := \{s : \|\ell^{(s)} - \ell_e\| \leq r_i\}$. Hence, the detection due to any $\mathcal{N}_j \subseteq \mathcal{N}(\ell_e)$ corresponds to the isolation of the event, and that due to $\mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)$ corresponds to false isolation. Note that in the case of Boolean sensing model $\mathcal{N}(\ell_e) = \mathcal{N}_i$.

In Section III, we formulate the problem of quickest detection of an event and *isolating the event to one of the influence subregions* $\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_N$ under a false alarm and false isolation constraint.

III. PROBLEM FORMULATION

We are interested in studying the *problem of distributed event detection/isolation* in the setting developed in Section II. Given a sample node deployment (i.e., given ℓ), and *having chosen a value of the detection range*, r_d , we partition the ROI, \mathcal{A} into the detection-subregions, $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$. Let \mathcal{N}_i be the set of sensors that detection-cover the region \mathcal{A}_i . Having chosen the influence range r_i , the influence region \mathcal{B}_i of the set of sensor nodes \mathcal{N}_i can be obtained. We define the following set of hypotheses

$$\mathbf{H}_0 : \text{event not occurred,}$$

$$\mathbf{H}_{T,i} : \text{event occurred at time } T \text{ in subregion } \mathcal{A}_i, \quad T = 1, 2, \dots, \quad i = 1, 2, \dots, N.$$

The event occurs in one of the detection subregions \mathcal{A}_i , but we will only be able to isolate it to one of the influence subregions \mathcal{B}_i that is consistent with the \mathcal{A}_i (see Section II-F). We study distributed procedures described by a stopping time τ , and an isolation decision $L(\tau) \in \{1, 2, \dots, N\}$ (i.e., the tuple (τ, L)) that detect an event at time τ and locate it to $L(\tau)$ (i.e., to the influence region $\mathcal{B}_{L(\tau)}$) subject

to a false alarm and false isolation constraint. The *false alarm constraint* considered is the average run length to false alarm ARL2FA, and the *false isolation constraint* considered is the probability of false isolation PFI, each of which we define as follows.

Definition 3 The **Average Run Length to False Alarm** ARL2FA of a change detection/isolation procedure τ is defined as the expected number of samples taken under null hypothesis \mathbf{H}_0 to raise an alarm, i.e.,

$$\text{ARL2FA}(\tau) := \mathbb{E}_\infty[\tau],$$

where $\mathbb{E}_\infty[\cdot]$ is the expectation operator (with the corresponding probability measure being $\mathbb{P}_\infty\{\cdot\}$) when the change occurs at infinity. ■

Definition 4 The **Probability of False Isolation** PFI of a change detection/isolation procedure τ is defined as the supremum of the probabilities of making an incorrect isolation decision, i.e.,

$$\text{PFI}(\tau) := \max_{1 \leq i \leq N} \sup_{\ell_e \in \mathcal{A}_i} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{L(\tau) = j\}$$

where we recall that $\mathcal{N}(\ell_e) = \{s : \|\ell^{(s)} - \ell_e\| \leq r_i\}$ is the set of sensors that influence covers $\ell_e \in \mathcal{A}_i$. ■

In the case of Boolean sensing model, the post-change pdfs depend only on the index i of the detection subregion where the event occurs, and hence, the PFI is given by

$$\text{PFI}(\tau) := \max_{1 \leq i \leq N} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \mathbb{P}_1^{(i)} \{L(\tau) = j\}.$$

In [2], Nikiforov defined the probability of false isolation, also, over the set of all possible change times, as $\text{SPFI}(\tau) := \sup_{1 \leq i \leq N} \sup_{1 \leq j \neq i \leq N} \sup_{T \geq 1} \mathbb{P}_T^{(i)} \{L(\tau) = j \mid \tau \geq T\}$. Define the following classes of change detection/isolation procedures,

$$\Delta(\gamma, \alpha) := \{(\tau, L) : \text{ARL2FA}(\tau) \geq \gamma, \text{SPFI}(\tau) \leq \alpha\},$$

$$\tilde{\Delta}(\gamma, \alpha) := \{(\tau, L) : \text{ARL2FA}(\tau) \geq \gamma, \text{PFI}(\tau) \leq \alpha\}.$$

We define the supremum average detection delay SADD performance for the procedure τ , in the same sense as Pollak [19] (also see [2]), as the maximum mean number of samples taken under any hypothesis $\mathbf{H}_{T,i}$, $i = 1, 2, \dots, N$, to raise an alarm, i.e.,

$$\text{SADD}(\tau) := \sup_{\ell_e \in \mathcal{A}} \sup_{T \geq 1} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} [\tau - T \mid \tau \geq T].$$

We are interested in obtaining an optimal procedure τ that minimises the SADD subject to the average run length to false alarm and the probability of false isolation constraints,

$$\begin{aligned} \inf \quad & \sup_{\ell_e \in \mathcal{A}} \sup_{T \geq 1} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} [\tau - T | \tau \geq T] \\ \text{subject to} \quad & \text{ARL2FA}(\tau) \geq \gamma \\ & \text{PFI}(\tau) \leq \alpha. \end{aligned}$$

The change detection/isolation problem that we pose here is motivated by the framework of [1], [2], [3], which we discuss in the next subsection.

A. Centralised Recursive Solution for the Boolean Sensing Model

In [2], Nikiforov and in [3], Tartakovsky studied a change detection/isolation problem that involves $N > 1$ post-change hypotheses (and one pre-change hypothesis). Thus, their formulation can be applied to our problem. But, in their model, the pdf of \mathbf{X}_k for $k \geq T$, under hypothesis $\mathbf{H}_{T,i}$, g_i is completely known. It should be noted that in our problem, in the case of power law path-loss sensing model, the pdf of the observations under any post-change hypothesis is unknown as the location of the event is unknown. The problem posed by Nikiforov [2] is

$$\inf_{(\tau, L) \in \Delta(\gamma, \alpha)} \sup_{1 \leq i \leq N} \sup \mathbb{E}_T^{(i)} [\tau - T | \tau \geq T], \quad (2)$$

and that by Tartakovsky [3] is

$$\inf_{(\tau, L) \in \tilde{\Delta}(\gamma, \alpha)} \sup_{1 \leq i \leq N} \sup \mathbb{E}_T^{(i)} [\tau - T | \tau \geq T]. \quad (3)$$

Nikiforov [2] and Tartakovsky [3] obtained asymptotically optimal *centralised change detection/isolation* procedures as $\min\{\gamma, \frac{1}{\alpha}\} \rightarrow \infty$, the SADD of which is given by the following theorem.

Theorem 1 (Nikiforov 03) *For the N -hypotheses change detection/isolation problem (for the Boolean sensing model) defined in Eqn. (2), the asymptotically maximum mean delay optimal detection/isolation procedure τ^* has the property,*

$$\text{SADD}(\tau^*) \lesssim \max \left\{ \frac{\ln \gamma}{\min_{1 \leq i \leq N} KL(g_i, g_0)}, \frac{-\ln(\alpha)}{\min_{1 \leq i \leq N, 1 \leq j \neq i \leq N} KL(g_i, g_j)} \right\}, \text{ as } \min \left\{ \gamma, \frac{1}{\alpha} \right\} \rightarrow \infty,$$

where $KL(\cdot, \cdot)$ is the Kullback–Leibler divergence function, and g_i is the pdf of the observation \mathbf{X}_k for $k \geq T$ under hypothesis $\mathbf{H}_{T,i}$. ■

Remark: Since, $\Delta(\gamma, \alpha) \subseteq \tilde{\Delta}(\gamma, \alpha)$, the asymptotic upper bound on SADD for τ^* is also an upper bound for the SADD over the set of procedures in $\tilde{\Delta}(\gamma, \alpha)$.

In the case of Boolean sensing model, for any post-change hypothesis $\mathbf{H}_{T,i}$, only the set of sensor nodes that detection cover (which is the same as influence cover) the subregion \mathcal{A}_i switch to a post-change pdf f_1 (and the distribution of other sensor nodes continues to be f_0). Since the pdf of the sensor observations are conditionally i.i.d., the pdf of the observation vector, in the Boolean sensing model, corresponds to the post-change pdf g_i of the centralised problem studied by Nikiforov [2] and by Tartakovsky [3]. Thus, their problem directly applies to our setting with the Boolean sensing model. In our work, however, we propose algorithms for the change detection/isolation problem for the power law sensing model as well. Also, the procedures proposed by Nikiforov and by Tartakovsky are (while being recursive) centralised, whereas we propose distributed procedures which are computationally simple.

In Section IV, we propose distributed detection/isolation procedures MAX, HALL and ALL and analyse their false alarm (ARL2FA), false isolation (PFI) and the detection delay (SADD) properties.

IV. DISTRIBUTED CHANGE DETECTION/ISOLATION PROCEDURES

In this section, we study the procedures MAX and ALL for change detection/isolation in a distributed setting. Also, we propose a distributed detection procedure “HALL,” and analyse the SADD, the ARL2FA, and the PFI performance.

A. The MAX Procedure

Tartakovsky and Veeravalli proposed a decentralised procedure MAX for a collocated scenario in [6]. We extend the MAX procedure to a large WSN under the ARL2FA and PFI constraints. Recalling Section II, each sensor node i employs CUSUM for local change detection between pdfs f_0 and $f_1(\cdot; r_d)$. Let $\tau^{(i)}$ be the random time at which the CUSUM statistic of sensor node i crosses the threshold c for the first time. At each time k , the local decision of sensor node i , $D_k^{(i)}$ is defined as

$$D_k^{(i)} := \begin{cases} 0, & \text{for } k < \tau^{(i)} \\ 1, & \text{for } k \geq \tau^{(i)}. \end{cases}$$

The global decision rule τ^{MAX} declares an alarm at the earliest time slot k at which all sensor nodes $j \in \mathcal{N}_i$ for some $i = 1, 2, \dots, N$ have crossed the threshold c . Thus,

$$\begin{aligned} \tau^{\text{MAX},(\mathcal{N}_i)} &:= \inf \left\{ k : D_k^{(j)} = 1, \forall j \in \mathcal{N}_i \right\} = \min \left\{ \tau^{(j)} : j \in \mathcal{N}_i \right\} \\ \tau^{\text{MAX}} &:= \min \left\{ \tau^{\text{MAX},(\mathcal{N}_i)} : 1 \leq i \leq N \right\}. \end{aligned}$$

i.e., the MAX procedure declares an alarm at the earliest time instant when the CUSUM statistic of all the sensor nodes \mathcal{N}_i corresponding to hypothesis $\mathbf{H}_{T,i}$ of some i have crossed the threshold at least once.

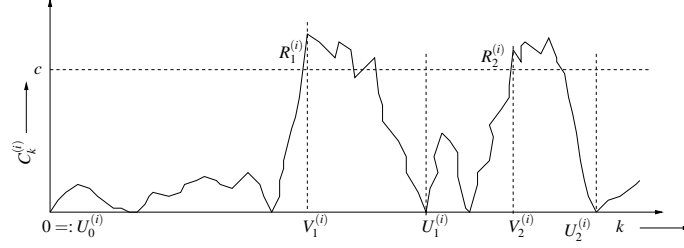


Fig. 3. ALL and HALL: Evolution of CUSUM statistic $C_k^{(i)}$ of node i plotted vs. k . Note that at time $k = V_j^{(i)}$, $R_j^{(i)}$ is the excess above the threshold.

The isolation rule is $L(\tau) = \arg \min_{1 \leq i \leq N} \{\tau^{\text{MAX}, (\mathcal{N}_i)}\}$, i.e., to declare that the event has occurred in the influence region $\mathcal{B}_{L(\tau)} = \mathcal{B}(\mathcal{N}_{L(\tau)})$ corresponding to the set of sensors $\mathcal{N}_{L(\tau)}$ that raised the alarm.

B. ALL Procedure

Mei, [9], and Tartakovsky and Kim, [25], proposed a decentralised procedure ALL, again for a collocated network. We extend the ALL procedure to a large extent network under the ARL2FA and the PFI constraints. Here, each sensor node i employs CUSUM for local change detection between pdfs f_0 and $f_1(\cdot; r_d)$. Let $C_k^{(i)}$ be the CUSUM statistic of sensor node i at time k . *The CUSUM in the sensor nodes is allowed to run freely even after crossing the threshold c .* Here, the local decision of sensor node i is

$$D_k^{(i)} := \begin{cases} 0, & \text{if } C_k^{(i)} < c \\ 1, & \text{if } C_k^{(i)} \geq c. \end{cases}$$

The global decision rule τ^{ALL} declares an alarm at the earliest time slot k at which the local decision of all the sensor nodes corresponding to a set \mathcal{N}_i , for some $i = 1, 2, \dots, N$, are 1, i.e.,

$$\begin{aligned} \tau^{\text{ALL}, (\mathcal{N}_i)} &:= \inf \left\{ k : D_k^{(j)} = 1, \forall j \in \mathcal{N}_i \right\} = \inf \left\{ k : C_k^{(j)} \geq c, \forall j \in \mathcal{N}_i \right\} \\ \tau^{\text{ALL}} &:= \min \left\{ \tau^{\text{ALL}, (\mathcal{N}_i)} : 1 \leq i \leq N \right\}. \end{aligned}$$

The isolation rule is $L(\tau) = \arg \min_{1 \leq i \leq N} \{\tau^{\text{ALL}, (\mathcal{N}_i)}\}$, i.e., to declare that the event has occurred in the influence region $\mathcal{B}_{L(\tau)} = \mathcal{B}(\mathcal{N}_{L(\tau)})$ corresponding to the set of sensors $\mathcal{N}_{L(\tau)}$ that raised the alarm.

C. HALL Procedure

Motivated by ALL, and the fact that sensor noise can make the CUSUM statistic fluctuate around the threshold, we propose a local decision rule which is 0 when the CUSUM statistic has visited zero and has not crossed the threshold yet and is 1 otherwise. We explain the HALL procedure below.

The following discussion is illustrated in Fig. 3. Each sensor node i computes a CUSUM statistic $C_k^{(i)}$ based on the LLR of its own observations between the pdfs $f_1(\cdot; r_d)$ and f_0 . Define $U_0^{(i)} := 0$. Define $V_1^{(i)}$ as the time at which $C_k^{(i)}$ crosses the threshold c (for the first time) as:

$$V_1^{(i)} := \inf \left\{ k : C_k^{(i)} \geq c \right\}$$

(see Fig. 3 where the “overshoots” $R_k^{(i)}$, at $V_k^{(i)}$, are also shown). Note that $\inf \emptyset := \infty$. Next define

$$U_1^{(i)} := \inf \left\{ k > V_1^{(i)} : C_k^{(i)} = 0 \right\}.$$

Now starting with $U_1^{(i)}$, we can recursively define $V_2^{(i)}, U_2^{(i)}$ etc. in the obvious manner (see Fig. 3). Each node i computes the local decision $D_k^{(i)}$ based on the CUSUM statistic $C_k^{(i)}$ as follows:

$$D_k^{(i)} = \begin{cases} 1, & \text{if } V_j^{(i)} \leq k < U_j^{(i)} \text{ for some } j \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The global decision rule is a stopping time τ^{HALL} defined as the earliest time slot k at which all the sensor nodes in a region have a local decision 1, i.e.,

$$\begin{aligned} \tau^{\text{HALL}, (\mathcal{N}_i)} &:= \inf \left\{ k : D_k^{(j)} = 1, \forall j \in \mathcal{N}_i \right\}, \\ \tau^{\text{HALL}} &:= \min \left\{ \tau^{\text{HALL}, (\mathcal{N}_i)} : 1 \leq i \leq N \right\}. \end{aligned}$$

The isolation rule is $L(\tau) = \arg \min_{1 \leq i \leq N} \{ \tau^{\text{HALL}, (\mathcal{N}_i)} \}$, i.e., to declare that the event has occurred in the influence region $\mathcal{B}_{L(\tau)} = \mathcal{B}(\mathcal{N}_{L(\tau)})$ corresponding to the set of sensors $\mathcal{N}_{L(\tau)}$ that raised the alarm.

Remark: The procedures HALL, MAX and ALL differ only in their local decision rule; the global decision rule as a function of $\{D_k^{(i)}\}$ is the same for HALL, MAX and ALL. For the distributed procedures MAX, ALL, and HALL, we analyse the ARL2FA in Section IV-D, the PFI in Section IV-E, and the SADD performance in Section IV-F.

D. Average Run Length to False Alarm (ARL2FA)

From the previous sections, we see that the stopping time of any procedure (MAX, ALL, or HALL) is the minimum of the stopping times corresponding to each \mathcal{N}_i , i.e.,

$$\tau^{\text{procedure}} := \min \left\{ \tau^{\text{procedure}, (\mathcal{N}_i)} : 1 \leq i \leq N \right\}.$$

Under the null hypothesis \mathbf{H}_0 , the CUSUM statistics $C_k^{(s)}$ s of sensors $s \in \mathcal{N}_i$ are driven by independent noise processes, and hence, $C_k^{(s)}$ s are independent. But, there can be a sensor that is common to two different \mathcal{N}_i s, and hence, $\tau^{\text{procedure}, (\mathcal{N}_i)}$ s, in general, are not independent. We provide asymptotic lower bounds for the ARL2FA for MAX, HALL, and ALL, in the following theorem.

Theorem 2 For local CUSUM threshold c ,

$$\text{ARL2FA}(\tau^{\text{MAX}}) \geq \exp(a_{\text{MAX}}c) \cdot (1 + o(1)) \quad (5)$$

$$\text{ARL2FA}(\tau^{\text{HALL}}) \geq \exp(a_{\text{HALL}}c) \cdot (1 + o(1)) \quad (6)$$

$$\text{ARL2FA}(\tau^{\text{ALL}}) \geq \exp(a_{\text{ALL}}c) \cdot (1 + o(1)) \quad (7)$$

($o(1) \rightarrow 0$ as $c \rightarrow \infty$), where for any arbitrarily small $\delta > 0$, $a_{\text{MAX}} = a_{\text{HALL}} = 1 - \delta$, $a_{\text{ALL}} = m - \delta$,

where $m = \min\{|\mathcal{N}_i \setminus \bigcup_{j \neq i, j \in \mathcal{I}} \mathcal{N}_j| : i \in \mathcal{I}\}$, \mathcal{I} is the set of indices of the detection sets that are minimal in the partially order of set inclusion among the detection sets.

Proof: See Appendix A. ■

Thus, for procedure, for a given ARL2FA requirement of γ , it is sufficient to choose the threshold c as

$$c = \frac{\ln \gamma}{a_{\text{procedure}}}(1 + o(1)), \quad \text{as } \gamma \rightarrow \infty. \quad (8)$$

E. Probability of False Isolation (PFI)

A false isolation occurs when the hypothesis $\mathbf{H}_{T,i}$ is true for some i and the hypothesis $\mathbf{H}_{T,j} \neq \mathbf{H}_{T,i}$ is declared to be true at the time of alarm, and the event does not lie in the region $\mathcal{B}(\mathcal{N}_j)$. The following theorem provide asymptotic upper bounds for the PFI for each of the procedures MAX, ALL, and HALL.

Theorem 3 For local CUSUM threshold c ,

$$\text{PFI}(\tau^{\text{MAX}}) \leq \frac{\exp(-b_{\text{MAX}}c)}{B_{\text{MAX}}} \cdot (1 + o(1)) \quad (9)$$

$$\text{PFI}(\tau^{\text{HALL}}) \leq \frac{\exp(-b_{\text{HALL}}c)}{B_{\text{HALL}}} \cdot (1 + o(1)) \quad (10)$$

$$\text{PFI}(\tau^{\text{ALL}}) \leq \frac{\exp(-b_{\text{ALL}}c)}{B_{\text{ALL}}} \cdot (1 + o(1)). \quad (11)$$

where $o(1) \rightarrow 0$ as $c \rightarrow \infty$, and $b_{\text{MAX}} = b_{\text{HALL}} = \frac{m\xi\omega_0}{2} - \frac{1+\bar{m}}{n}$, $b_{\text{ALL}} = \frac{m\xi\omega_0}{2} - \frac{1}{n}$, $\omega_0 = 1$ for Boolean sensing model, ξ is 2 for Boolean sensing model and is 1 for path-loss sensing model, $m = \min\{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)| : 1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)\}$ and $\bar{m} = \max\{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)| : 1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)\}$, and B_{MAX} , B_{HALL} , and B_{ALL} are positive constants.

Proof: See Appendix B. ■

Thus, for a given PFI requirement of α , the threshold c for should satisfy

$$c = \frac{-\ln B_{\text{procedure}} - \ln \alpha}{b_{\text{procedure}}}(1 + o(1)) = \frac{-\ln \alpha}{b_{\text{procedure}}}(1 + o(1)), \quad \text{as } \alpha \rightarrow 0. \quad (12)$$

F. Supremum Average Detection Delay (SADD)

In this section, we analyse the SADD performance of the distributed detection/isolation procedures. We observe that for any sample path of the observation process, for the same threshold c , the MAX rule raises an alarm first, followed by the HALL rule, and then by the ALL rule. This ordering is due to the following reason. For each sensor node s , let $\tau^{(s)}$ be the *first time instant* at which the CUSUM statistic $C_k^{(s)}$ crosses the threshold c (denoted by $V_1^{(i)}$ in Figure 3). Before time $\tau^{(s)}$, the local decision is 0 for all the procedures, MAX, ALL, and HALL. For MAX, for all $k \geq \tau^{(s)}$, the local decision $D_k^{(s)} = 1$. Thus, the stopping time of MAX is at least as early as that of HALL and ALL. The local decision of ALL is 1 ($D_k^{(s)} = 1$) only at those times k for which $C_k^{(s)} \geq c$. However, even when $C_k^{(s)} < c$, the local decision of HALL is 1 if $V_j^{(s)} \leq k < U_j^{(s)}$ (see Figure 3) for some j . Thus, the local decisions of MAX, HALL, and ALL are ordered as, for all $k \geq 1$, $D_k^{(s)}(\text{MAX}) \geq D_k^{(s)}(\text{HALL}) \geq D_k^{(s)}(\text{ALL})$, and hence, $\tau^{\text{MAX},(\mathcal{N}_i)} \leq \tau^{\text{HALL},(\mathcal{N}_i)} \leq \tau^{\text{ALL},(\mathcal{N}_i)}$. Each of the stopping times MAX, HALL, or ALL is the minimum of stopping times corresponding to the sets of sensors $\{\mathcal{N}_i : i = 1, 2, \dots, N\}$, i.e.,

$$\tau^{\text{procedure}} = \min\{\tau^{\text{procedure},(\mathcal{N}_i)} : i = 1, 2, \dots, N\}$$

where “procedure” can be MAX or HALL or ALL. Hence, we have

$$\tau^{\text{MAX}} \leq \tau^{\text{HALL}} \leq \tau^{\text{ALL}}. \quad (13)$$

From [9], we see that

$$\sup_{T \geq 1} \mathbb{E}_T^{(i)} \left[\tau^{\text{ALL},(\mathcal{N}_i)} - T \mid \tau^{\text{ALL},(\mathcal{N}_i)} \geq T \right] = \frac{c}{I} (1 + o(1)) \quad (14)$$

where I is the Kullback–Leibler divergence between the post-change and the pre-change pdfs. For $\ell_e \in \mathcal{A}_i$, we have $\forall s \in \mathcal{N}_i$, $d_{e,s} \leq r_d$. Also, since $\tau^{\text{ALL}} \leq \tau^{\text{ALL},(\mathcal{N}_i)}$, we have

$$\sup_{\ell_e \in \mathcal{A}_i} \sup_{T \geq 1} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left[\tau^{\text{ALL}} - T \mid \tau^{\text{ALL}} \geq T \right] \leq \sup_{\ell_e \in \mathcal{A}_i} \sup_{T \geq 1} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left[\tau^{\text{ALL},(\mathcal{N}_i)} - T \mid \tau^{\text{ALL}} \geq T \right] \quad (15)$$

From Appendix C, Eqn. (15) becomes,

$$\begin{aligned} \sup_{\ell_e \in \mathcal{A}_i} \sup_{T \geq 1} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left[\tau^{\text{ALL}} - T \mid \tau^{\text{ALL}} \geq T \right] &\leq \sup_{\ell_e \in \mathcal{A}_i} \sup_{T \geq 1} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left[\tau^{\text{ALL},(\mathcal{N}_i)} - T \mid \tau^{\text{ALL},(\mathcal{N}_i)} \geq T \right] \\ &= \frac{c}{\text{KL}(f_1(\cdot; r_d), f_0)} (1 + o(1)) \end{aligned} \quad (16)$$

From the above equation, and from Eqn. (13), we have

$$\text{SADD}(\tau^{\text{MAX}}) \leq \text{SADD}(\tau^{\text{HALL}}) \leq \text{SADD}(\tau^{\text{ALL}}) \leq \frac{c}{\text{KL}(f_1(\cdot; r_d), f_0)} (1 + o(1)), \text{ as } c \rightarrow \infty, \quad (17)$$

Remark: Recall from Section II-B that $\mu_1 = h_e \rho(r_d)$. We now see that μ_1 governs the detection delay performance, and μ_1 can be chosen such that a requirement on SADD is met. Thus, to achieve a requirement on SADD, we need to choose r_d appropriately. A small value of r_d (gives a large μ_1 and hence,) gives less detection delay compared to a large value of r_d . But, a small r_d requires more sensors to detection-cover the ROI.

In the next subsection, we discuss the asymptotic minimax delay optimality of the distributed procedures in relation to Theorem 1.

G. Asymptotic Upper Bound on SADD

For any change detection/isolation procedure to achieve a ARL2FA requirement of γ and PFI requirement of α , a threshold c is chosen such that it satisfies Eqns. 8 and 12, i.e.,

$$c = \max \left\{ \frac{\ln \gamma}{a_{\text{procedure}}}, \frac{-\ln \alpha}{b_{\text{procedure}}} \right\} (1 + o(1)). \quad (18)$$

Therefore, from Eqn.(17), the SADD is given by

$$\text{SADD}(\tau^{\text{procedure}}) \leq \frac{1}{\text{KL}(f_1(\cdot; r_d), f_0)} \cdot \max \left\{ \frac{\ln \gamma}{a_{\text{procedure}}}, \frac{-\ln \alpha}{b_{\text{procedure}}} \right\} (1 + o(1)). \quad (19)$$

where $o(1) \rightarrow 0$ as $\min\{\gamma, \frac{1}{\alpha}\} \rightarrow \infty$. Note that as r_d decreases, $\text{KL}(f_1(\cdot; r_d), f_0) = \frac{h_e^2 \rho(r_d)^2}{2\sigma^2}$ increases. Thus, to achieve a smaller detection delay, the detection range r_d can be decreased, and the number of sensors n can be increased to cover the ROI.

We can compare the asymptotic SADD performance of the distributed procedures HALL, MAX and ALL against Theorem 1 for the Boolean sensing model. For Gaussian pdfs f_0 and f_1 , the KL divergence between the hypotheses $\mathbf{H}_{T,i}$ and $\mathbf{H}_{T,j}$ is given by

$$\begin{aligned} \text{KL}(g_i, g_j) &= \int \ln \left(\frac{\prod_{s \in \mathcal{N}_i} f_1(x^{(s)}) \prod_{s' \notin \mathcal{N}_i} f_0(x^{(s')})}{\prod_{s \in \mathcal{N}_j} f_1(x^{(s)}) \prod_{s' \notin \mathcal{N}_j} f_0(x^{(s')})} \right) \prod_{s \in \mathcal{N}_i} f_1(x^{(s)}) \prod_{s' \notin \mathcal{N}_i} f_0(x^{(s')}) d\mathbf{x} \\ &= \int \left(\ln \left(\prod_{s \in \mathcal{N}_i} \frac{f_1(x^{(s)})}{f_0(x^{(s)})} \right) - \ln \left(\prod_{s \in \mathcal{N}_j} \frac{f_1(x^{(s)})}{f_0(x^{(s)})} \right) \right) \prod_{s \in \mathcal{N}_i} f_1(x^{(s)}) \prod_{s' \notin \mathcal{N}_i} f_0(x^{(s')}) d\mathbf{x} \\ &= \sum_{s \in \mathcal{N}_i} \text{KL}(f_1, f_0) - \sum_{s \in \mathcal{N}_j \cap \mathcal{N}_i} \text{KL}(f_1, f_0) + \sum_{s \in \mathcal{N}_j \setminus \mathcal{N}_i} \text{KL}(f_1, f_0) \\ &= |\mathcal{N}_i \Delta \mathcal{N}_j| \text{KL}(f_1, f_0) \end{aligned}$$

where the operator Δ represents the symmetric difference between the sets. Thus, from Theorem 1 for

Gaussian f_0 and f_1 , we have

$$\begin{aligned} \text{SADD}(\tau^*) &\leq \frac{1}{\text{KL}(f_1, f_0)} \cdot \max \left\{ \frac{\ln \gamma}{a^*}, \frac{-\ln \alpha}{b^*} \right\} (1 + o(1)), \\ \text{where } a^* &= \min_{1 \leq i \leq N} |\mathcal{N}_i|, \\ \text{and } b^* &= \min_{\substack{1 \leq i \leq N \\ 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i}} |\mathcal{N}_i \Delta \mathcal{N}_j|. \end{aligned}$$

The SADD performance of the distributed procedure with the Boolean sensing model is

$$\text{SADD}(\tau^{\text{procedure}}) \leq \frac{1}{\text{KL}(f_1, f_0)} \cdot \max \left\{ \frac{\ln \gamma}{a_{\text{procedure}}}, \frac{-\ln \alpha}{b_{\text{procedure}}} \right\} (1 + o(1)). \quad (20)$$

where $o(1) \rightarrow 0$ as $\min\{\gamma, \frac{1}{\alpha}\} \rightarrow \infty$. Thus, the asymptotically optimal upper bound on SADD (which corresponds to the optimum centralised procedure τ^*) and that of the distributed procedures ALL, HALL, and MAX scale in the same way as $\ln \gamma / \text{KL}(f_1, f_0)$ and $-\ln \alpha / \text{KL}(f_1, f_0)$.

V. NUMERICAL RESULTS

We consider a deployment of 7 nodes with the detection range $r_d = 1$, in a hexagonal ROI (see Fig. 4) such that we get $N = 12$ detection subregions, and $\mathcal{N}_1 = \{1, 3, 4, 6\}$, $\mathcal{N}_2 = \{1, 3, 4\}$, $\mathcal{N}_3 = \{1, 2, 3, 4\}$, $\mathcal{N}_4 = \{1, 2, 4\}$, $\mathcal{N}_5 = \{1, 2, 4, 5\}$, $\mathcal{N}_6 = \{2, 4, 5\}$, $\mathcal{N}_7 = \{2, 4, 5, 7\}$, $\mathcal{N}_8 = \{4, 5, 7\}$, $\mathcal{N}_9 = \{4, 5, 6, 7\}$, $\mathcal{N}_{10} = \{4, 6, 7\}$, $\mathcal{N}_{11} = \{3, 4, 6, 7\}$, and $\mathcal{N}_{12} = \{3, 4, 6\}$. The pre-change pdf considered is $f_0 \sim \mathcal{N}(0, 1)$, and the detection range and the influence range considered are $r_d = 1.0$ and $r_i = 1.5$ respectively.

We compute the SADD, the ARL2FA and the PFI performance of MAX, HALL, ALL, and Nikiforov's procedure ([2]) for the Boolean sensing model with $f_1 \sim \mathcal{N}(1, 1)$, and plot the SADD vs $\log(\text{ARL2FA})$ performance in Fig. 5(a), of the change detection/isolation procedures for $\text{PFI} \leq 5 \times 10^{-2}$. The local CUSUM threshold c that yields the target ARL2FA and other simulation parameters and results are tabulated in Table I. To obtain the SADD the event is assumed to occur at time 1, which corresponds to the maximum mean delay (see [19], [26]). We observe from Fig. 5(a) that the SADD performance of MAX is the worst and that of Nikiforov's is the best. Also, we note that the performance of the distributed procedures, ALL and HALL, are very close to that of the optimal centralised procedure. For eg., for a requirement of $\text{ARL2FA} = 10^5$ (and $\text{PFI} \leq 5 \times 10^{-2}$), we observe from Fig. 5(a) that $\text{SADD}(\tau^{\text{MAX}}) = 26.43$, $\text{SADD}(\tau^{\text{HALL}}) = 13.78$, $\text{SADD}(\tau^{\text{ALL}}) = 12.20$, and $\text{SADD}(\tau^*) = 11.28$. Since MAX does not make use of the dynamics of $C_k^{(s)}$ beyond τ^s , its SADD vs ARL2FA performance is poor. On the other hand, ALL and HALL make use of $C_k^{(s)}$ for all k and hence, give a better performance.

For the same sensor deployment in Fig. 4, we compute the SADD and the ARL2FA for the square law path loss ($\eta = 2$) sensing model given in Section II. Also, the signal strength h_e is taken to be unity.

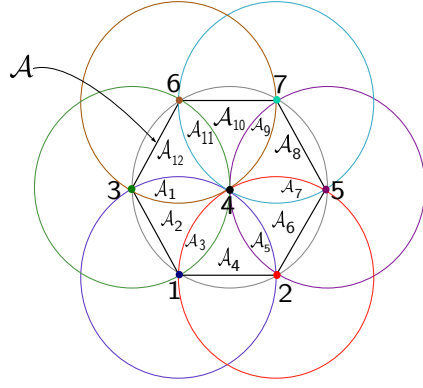
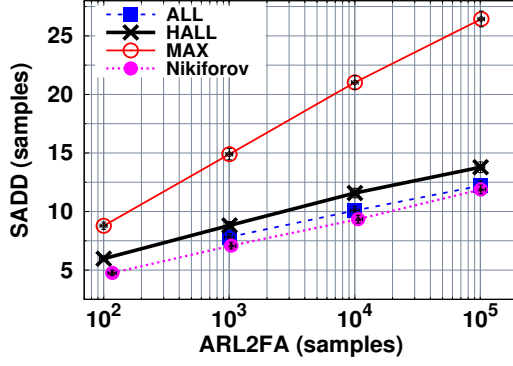


Fig. 4. **Sensor nodes placement:** 7 sensor nodes (which are numbered $1, 2, \dots, 7$) represented by small filled circles are placed in the hexagonal ROI \mathcal{A} . The sensor nodes partition the ROI into the detection subregions $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_{12}$ (for both the Boolean and the power law path loss sensing models).

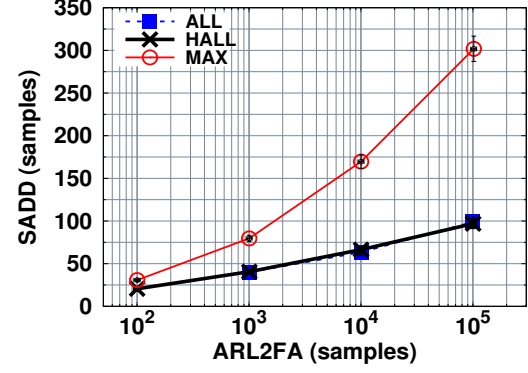
TABLE I

SIMULATION PARAMETERS AND RESULTS FOR THE BOOLEAN SENSING MODEL FOR $\text{PFI} \leq 5 \times 10^{-2}$

Detection/ Isolation procedure	No. of MC runs	Threshold c	ARL2FA	99% Confidence interval		SADD	99% Confidence interval	
				ARL2FA _{lower}	ARL2FA _{upper}		SADD _{lower}	SADD _{upper}
MAX	10^4	2.71	10^2	93.69	106.61	8.77	8.45	9.09
	10^4	4.93	10^3	942.10	1065.81	14.89	14.41	15.37
	10^4	7.24	10^4	9398.61	10640.99	21.01	20.42	21.61
	10^4	9.52	10^5	95696.90	108008.89	26.43	25.76	27.11
HALL	10^4	1.67	10^2	92.67	107.58	5.96	5.72	6.20
	10^4	2.69	10^3	927.17	1085.48	8.81	8.48	9.14
	10^4	3.66	10^4	9239.97	10826.71	11.58	11.17	11.99
	10^4	4.52	10^5	92492.85	108389.15	13.78	13.32	14.23
ALL	10^4	2.16	10^3	915.94	1089.33	7.82	7.53	8.11
	10^4	2.96	10^4	9197.23	10811.90	10.07	9.70	10.44
	10^4	3.71	10^5	92205.45	107952.43	12.20	11.76	12.63
Nikiforov	10^4	2.75	10^2	98.30	116.32	4.75	4.52	4.98
	10^4	4.50	10^3	986.48	1048.23	7.08	6.79	7.38
	10^4	6.32	10^4	9727.19	10261.94	9.14	9.00	9.68
	10^4	8.32	10^5	98961.41	110415.50	11.28	11.00	12.25



(a) SADD vs ARL2FA for the Boolean model



(b) SADD vs ARL2FA for the square law path loss model

Fig. 5. SADD versus ARL2FA (for $\text{PFI} \leq 5 \times 10^{-2}$) for MAX, HALL, ALL and Nikiforov's procedure for the Boolean and the square law path loss sensing models. In the Boolean sensing model, the system parameters are $f_0 \sim N(0, 1)$, $f_1 \sim N(0, 1)$, and in the case of path loss sensing model, the parameters are $f_0 \sim N(0, 1)$, $h_e = 1$, $r_d = 1.0$, $r_i = 1.5$.

TABLE II

SIMULATION PARAMETERS AND RESULTS FOR THE SQUARE LAW PATH LOSS SENSING MODEL FOR $\text{PFI} \leq 5 \times 10^{-2}$

Detection/ Isolation procedure	No. of MC runs	Threshold c	ARL2FA	99% Confidence interval		SADD	99% Confidence interval	
				ARL2FA _{lower}	ARL2FA _{upper}		SADD _{lower}	SADD _{upper}
MAX	10^4	2.71	10^2	93.69	106.61	30.74	29.31	32.17
	10^4	4.93	10^3	942.10	1065.81	79.60	75.86	83.34
	10^4	7.23	10^4	9398.61	10640.99	169.63	161.61	177.65
	10^4	9.52	10^5	95696.90	108008.89	301.77	286.88	316.66
HALL	10^4	1.67	10^2	92.67	107.58	20.58	19.43	21.74
	10^4	2.69	10^3	927.17	1085.48	40.56	38.24	42.88
	10^4	3.66	10^4	9239.97	10826.71	66.45	62.57	70.33
	10^4	4.52	10^5	92492.85	108389.15	96.93	91.03	102.82
ALL	10^4	1.33	10^2	92.24	107.79	20.19	19.06	21.32
	10^4	2.16	10^3	915.94	1089.33	39.90	37.59	42.21
	10^4	2.96	10^4	9197.23	10811.90	63.34	59.43	67.24
	10^4	3.71	10^5	92205.45	107952.43	98.96	93.01	104.92

Thus, the sensor sets (\mathcal{N}_i s) and the detection subregions (\mathcal{A}_i s) are the same as in the Boolean model, we described above. Since r_d is taken as 1, $f_1(\cdot; r_d) \sim \mathcal{N}(1, 1)$. Thus, the LLR of observation $X_k^{(s)}$ is given by $\ln \left(\frac{f_1(X_k^{(s)}; r_d)}{f_0(X_k^{(s)})} \right) = X_k^{(s)} - \frac{1}{2}$, which is the same as that in the Boolean sensing model. Hence, under the event not occurred hypothesis, the ARL2FA under the path loss sensing model is the same as that of the

Boolean sensing model. The CUSUM threshold c that yields the target ARL2FAs and other parameters and results are tabulated in Table II. To obtain the SADD the event is assumed to occur at time 1, and at a distance of r_i from all the nodes of \mathcal{N}_i that influence covers the event (which corresponds to the maximum detection delay). We plot the SADD vs $\log(\text{ARL2FA})$ in Fig. 5(b). The ordering on SADD for any ARL2FA across the procedures is the same as that in the Boolean model, and can be explained in the same manner. The ambiguity in ℓ_e affects $f_1(\cdot; d_{e,s})$ and shows up as large SADD values.

VI. CONCLUSION

We consider the quickest distributed event detection/isolation problem in a large extent WSN with a practical sensing model which incorporates the reduction in signal strength with distance. We formulate the change detection/isolation problem in the optimality framework of [2] and [3]. We propose distributed detection/isolation procedures, MAX, ALL and HALL and show that as $\min\{\text{ARL2FA}, 1/\text{PFI}\} \rightarrow \infty$, the SADD performance of the distributed procedures grows in the same scale as that of the optimal centralised procedure of Tartakovsky [3] and Nikiforov [2].

APPENDIX A

PROOF OF THEOREM 2

From detection sensor sets $\mathcal{N}_i, i = 1, 2, \dots, N$, we choose the collection of indices $\mathcal{I} \subseteq \{1, 2, \dots, N\}$ such that any two sensor sets $\mathcal{N}_i, \mathcal{N}_j, i, j \in \mathcal{I}$, are not partially ordered by set inclusion. For each $i \in \mathcal{I}$, define the set of sensors that are unique to the sensor set \mathcal{N}_i , $\mathcal{M}_i := \mathcal{N}_i \setminus \bigcup_{j \neq i, j \in \mathcal{I}} \mathcal{N}_j \subseteq \mathcal{N}_i$. The sets $\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_{|\mathcal{I}|}$ are disjoint. Under the null hypothesis, \mathbf{H}_0 , the observations of sensors in the sensor sets $\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_{|\mathcal{I}|}$ are iid, with the pdf $f_0 \sim \mathcal{N}(0, \sigma^2)$. For every \mathcal{N}_i , there exists \mathcal{M}_j such that $\mathcal{M}_j \subseteq \mathcal{N}_i$, so that $\tau^{\text{rule}, (\mathcal{N}_i)} \geq \tau^{\text{rule}, (\mathcal{M}_j)}$. Hence, $\tau^{\text{procedure}} = \min\{\tau^{\text{procedure}, (\mathcal{N}_i)} : i = 1, 2, \dots, N\} \geq \min\{\tau^{\text{procedure}, (\mathcal{M}_i)} : i \in \mathcal{I}\} =: \hat{\tau}^{\text{rule}}$. Hence,

$$\begin{aligned} \mathbb{E}_\infty [\tau^{\text{rule}}] &\geq \mathbb{E}_\infty [\hat{\tau}^{\text{rule}}] \geq e^{mc} \cdot \mathbb{P} \left\{ \hat{\tau}^{\text{rule}} > e^{mc} \right\} \quad (\text{by the Markov inequality}) \\ \text{or, } \frac{\mathbb{E}_\infty [\tau^{\text{rule}}]}{e^{mc}} &\geq \mathbb{P} \left\{ \hat{\tau}^{\text{rule}} > e^{mc} \right\} = \prod_{i \in \mathcal{I}} \mathbb{P}_\infty \left\{ \tau^{\text{rule}, (\mathcal{M}_i)} > e^{mc} \right\}. \end{aligned} \quad (21)$$

We analyse $\mathbb{P}_\infty \left\{ \tau^{\text{rule}, (\mathcal{M}_i)} > e^{mc} \right\}$ as $c \rightarrow \infty$, for ALL, MAX, and HALL. For ALL,

$$\begin{aligned} \mathbb{P}_\infty \left\{ \tau^{\text{ALL}, (\mathcal{M}_i)} = k \right\} &\leq \mathbb{P}_\infty \left\{ C_k^{(s)} \geq c, \forall s \in \mathcal{M}_i \right\} = \prod_{s \in \mathcal{M}_i} \mathbb{P}_\infty \left\{ C_k^{(s)} \geq c \right\} \\ &\leq e^{-cm_i} \quad (\text{using Wald's inequality}) \end{aligned}$$

$$\begin{aligned} \text{Therefore, } \mathbb{P}_\infty \left\{ \tau^{\text{ALL}, (\mathcal{M}_i)} \leq k \right\} &\leq k \cdot e^{-cm_i} \\ \mathbb{P}_\infty \left\{ \tau^{\text{ALL}, (\mathcal{M}_i)} > e^{mc} \right\} &\geq 1 - e^{-c(m_i - m)}. \end{aligned}$$

Hence, for any $m < m_i$, we have $\liminf_{c \rightarrow \infty} \mathbb{P}_\infty \{ \tau^{\text{ALL}, (\mathcal{M}_i)} > e^{mc} \} = 1$. A large m (which is smaller than all m_i s) is desirable. Thus, a good choice for m is $a_{\text{ALL}} = \min\{m_i : i \in \mathcal{I}\} - \delta$, for some arbitrarily small $\delta > 0$. Hence, from Eqn. (21),

$$\mathbb{E}_\infty [\tau^{\text{ALL}}] \geq \exp(a_{\text{ALL}} c) (1 + o(1)) \quad (22)$$

For MAX, at the stopping time of MAX, at least one of the CUSUM statistics is above the threshold c ,

$$\begin{aligned} \mathbb{P}_\infty \{ \tau^{\text{MAX}, (\mathcal{M}_i)} = k \} &\leq \mathbb{P}_\infty \{ C_k^{(s)} \geq c, \text{ for some } s \in \mathcal{M}_i \} \\ &\leq \sum_{s \in \mathcal{M}_i} \mathbb{P}_\infty \{ C_k^{(s)} \geq c \} \\ &\leq m_i e^{-c} \quad (\text{using Wald's inequality}). \end{aligned} \quad (23)$$

$$\begin{aligned} \text{Therefore, for any arbitrarily small } \delta > 0, \mathbb{P}_\infty \{ \tau^{\text{MAX}, (\mathcal{M}_i)} > e^{(1-\delta)c} \} &\geq 1 - m_i e^{-\delta c} \\ \liminf_{c \rightarrow \infty} \mathbb{P}_\infty \{ \tau^{\text{MAX}, (\mathcal{M}_i)} > e^{(1-\delta)c} \} &= 1. \end{aligned} \quad (24)$$

Let $a_{\text{MAX}} = 1 - \delta$. For any arbitrarily small $\delta > 0$, we see from Eqn. (21),

$$\mathbb{E}_\infty [\tau^{\text{MAX}}] \geq \exp((1 - \delta)c) (1 + o(1)) =: \exp(a_{\text{MAX}} c) (1 + o(1)), \quad (25)$$

For HALL, for the same threshold c , the stopping time of HALL is after that of MAX. Hence, $\tau^{\text{HALL}} \geq \tau^{\text{MAX}}$. Hence, $\mathbb{E}_\infty [\tau^{\text{HALL}}] \geq \mathbb{E}_\infty [\tau^{\text{MAX}}] \geq \exp((1 - \delta)c) (1 + o(1))$ (from Eqn. (25)). Thus, for $a_{\text{ALL}} := 1 - \delta$, for any arbitrarily small $\delta > 0$,

$$\mathbb{E}_\infty [\tau^{\text{HALL}}] \geq \exp(a_{\text{ALL}} c) (1 + o(1)) \quad (26)$$

APPENDIX B

PROOF OF THEOREM 3

Consider $\ell_e \in \mathcal{A}_i$. The probability of false isolation when the detection is due to $\mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)$ is

$$\begin{aligned} \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}} = \tau^{\text{rule}, (\mathcal{N}_j)} \} &= \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}, (\mathcal{N}_j)} \leq \tau^{\text{rule}, (\mathcal{N}_h)}, \forall h = 1, 2, \dots, N \} \\ &\leq \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}, (\mathcal{N}_j)} \leq \tau^{\text{rule}, (\mathcal{N}_i)} \} \\ &= \sum_{k=1}^{\infty} \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}, (\mathcal{N}_i)} = k \} \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}, (\mathcal{N}_j)} \leq k \mid \tau^{\text{rule}, (\mathcal{N}_i)} = k \} \\ &= \sum_{k=1}^{\infty} \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}, (\mathcal{N}_i)} = k \} \left[\sum_{t=1}^k \mathbb{P}_1^{(\mathbf{d}(\ell_e))} \{ \tau^{\text{rule}, (\mathcal{N}_j)} = t \mid \tau^{\text{rule}, (\mathcal{N}_i)} = k \} \right] \end{aligned}$$

A. $\text{PFI}(\tau^{\text{ALL}})$ – *Boolean Sensing Model*

$$\begin{aligned} \mathbf{P}_1^{(i)} \left\{ \tau^{\text{ALL},(\mathcal{N}_j)} = t \mid \tau^{\text{ALL},(\mathcal{N}_i)} = k \right\} &\leq \mathbf{P}_1^{(i)} \left\{ C_t^{(s)} \geq c, \forall s \in \mathcal{N}_j \mid \tau^{\text{ALL},(\mathcal{N}_i)} = k \right\} \\ &\leq \mathbf{P}_\infty \left\{ C_t^{(s)} \geq c, \forall s \in \mathcal{N}_j \setminus \mathcal{N}_i \right\} \\ &\leq \exp(-|\mathcal{N}_j \setminus \mathcal{N}_i|c) \quad (\text{using Wald's inequality}). \end{aligned}$$

$$\begin{aligned} \text{Therefore, } \mathbf{P}_1^{(i)} \left\{ \tau^{\text{ALL},(\mathcal{N}_j)} \leq \tau^{\text{ALL},(\mathcal{N}_i)} \right\} &\leq \exp(-|\mathcal{N}_j \setminus \mathcal{N}_i|c) \cdot \mathbf{E}_1^{(i)} \left[\tau^{\text{ALL},(\mathcal{N}_i)} \right] \\ &\leq \exp(-(|\mathcal{N}_j \setminus \mathcal{N}_i|c - \ln(c))) \cdot \frac{1}{\alpha|\mathcal{N}_i|} (1 + o(1)). \end{aligned}$$

$$\begin{aligned} \text{Hence, } \text{PFI}(\tau^{\text{ALL}}) &\leq \max_{1 \leq i \leq N} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \mathbf{P}_1^{(i)} \left\{ \tau^{\text{ALL},(\mathcal{N}_j)} \leq \tau^{\text{ALL},(\mathcal{N}_i)} \right\} \\ &\leq \frac{\exp(-(mc - \ln(c)))}{\underline{n}\alpha} (1 + o(1)) \end{aligned} \quad (27)$$

where $\underline{n} = \min\{|\mathcal{N}_i| : i = 1, 2, \dots, N\}$, $m = \min_{1 \leq i \leq N, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \{|\mathcal{N}_j \setminus \mathcal{N}_i|\}$. For any n , there exists $c_0(n)$ such that for all $c > c_0(n)$, $c < e^{c/n}$. Using this inequality, for sufficiently large c

$$\text{PFI}(\tau^{\text{ALL}}) \leq \frac{\exp(-(m - \frac{1}{n})c)}{\underline{n}\alpha} (1 + o(1)) = \frac{\exp(-b_{\text{ALL}} \cdot c)}{B_{\text{ALL}}} (1 + o(1)),$$

where $b_{\text{ALL}} = m - 1/n$ and $B_{\text{ALL}} = \underline{n}\alpha$.

B. $\text{PFI}(\tau^{\text{MAX}})$ – *Boolean Sensing Model*

$$\begin{aligned} \mathbf{P}_1^{(i)} \left\{ \tau^{\text{MAX},(\mathcal{N}_j)} = t \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} &\leq \mathbf{P}_1^{(i)} \left\{ \tau^{(s)} \leq t, \forall s \in \mathcal{N}_j \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} \\ &\leq \mathbf{P}_\infty \left\{ \tau^{(s)} \leq t, \forall s \in \mathcal{N}_j \setminus \mathcal{N}_i \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} \\ &= \prod_{s \in \mathcal{N}_j \setminus \mathcal{N}_i} \sum_{n=1}^t \mathbf{P}_\infty \left\{ \tau^{(s)} = n \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} \\ &= \prod_{s \in \mathcal{N}_j \setminus \mathcal{N}_i} \sum_{n=1}^t \mathbf{P}_\infty \left\{ C_n^{(s)} \geq c \right\} \\ &\leq \exp(-m_{ji}c) t^{m_{ji}} \end{aligned}$$

$$\begin{aligned} \text{Hence, } \mathbf{P}_1^{(i)} \left\{ \tau^{\text{MAX},(\mathcal{N}_j)} \leq \tau^{\text{MAX},(\mathcal{N}_i)} \right\} &\leq \exp(-m_{ji}c) \mathbf{E}_1^{(i)} \left[(\tau^{\text{MAX},(\mathcal{N}_i)})^{1+m_{ji}} \right] \\ &\leq \exp(-m_{ji}c) \frac{c^{1+m_{ji}}}{\alpha^{1+m_{ji}}} (1 + o(1)) \\ &= \frac{\exp(-(m_{ji}c - (1 + m_{ji})\ln(c)))}{\alpha^{1+m_{ji}}} (1 + o(1)) \end{aligned}$$

Let $m = \min_{1 \leq i \leq N, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} m_{ji}$, $\bar{m} = \max_{1 \leq i \leq N, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} m_{ji}$, and $\alpha^* = \min_{1 \leq i \leq N, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \alpha^{1+m_{ji}}$.

$$\begin{aligned} \text{Therefore, } \text{PFI}(\tau^{\text{MAX}}) &\leq \max_{1 \leq i \leq N} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \text{P}_1^{(i)} \left\{ \tau^{\text{MAX}, (\mathcal{N}_j)} \leq \tau^{\text{MAX}, (\mathcal{N}_i)} \right\} \\ &\leq \frac{\exp(-(mc - (1 + \bar{m}) \ln(c)))}{\alpha^*} (1 + o(1)). \end{aligned}$$

For any n , there exists $c_0(n)$ such that for all $c > c_0(n)$, $c < e^{c/n}$. Hence, for sufficiently large c

$$\begin{aligned} \text{PFI}(\tau^{\text{MAX}}) &\leq \max_{1 \leq i \leq N} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \text{P}_T^{(i)} \left\{ \tau^{\text{MAX}, (\mathcal{N}_j)} \leq \tau^{\text{MAX}, (\mathcal{N}_i)} \right\} \\ &\leq \frac{\exp(-(m - \frac{1+\bar{m}}{n})c)}{\alpha^*} (1 + o(1)) = \frac{\exp(-b_{\text{MAX}} \cdot c)}{B_{\text{MAX}}} (1 + o(1)), \end{aligned}$$

where $b_{\text{MAX}} = m - ((1 + \bar{m})/n)$ and $B_{\text{MAX}} = \alpha^*$.

C. $\text{PFI}(\tau^{\text{HALL}})$ – Boolean Sensing Model

$$\text{P}_1^{(i)} \left\{ \tau^{\text{HALL}, (\mathcal{N}_j)} = t \mid \tau^{\text{HALL}, (\mathcal{N}_i)} = k \right\} \leq \text{P}_1^{(i)} \left\{ \tau^{(s)} \leq t, \forall s \in \mathcal{N}_j \mid \tau^{\text{HALL}, (\mathcal{N}_i)} = k \right\}$$

which has the same form as that of MAX. Hence, from the analysis of MAX, it follows that

$$\begin{aligned} \text{P}_1^{(i)} \left\{ \tau^{\text{HALL}, (\mathcal{N}_j)} \leq \tau^{\text{HALL}, (\mathcal{N}_i)} \right\} &\leq \exp(-m_{ji}c) \text{E}_1^{(i)} \left[(\tau^{\text{HALL}, (\mathcal{N}_i)})^{1+m_{ji}} \right] \\ &\leq \exp(-m_{ji}c) \frac{c^{1+m_{ji}}}{|\mathcal{N}_i|^{1+m_{ji}} \alpha^{1+m_{ji}}} (1 + o(1)) \\ &= \exp(-(m_{ji}c - (1 + m_{ji}) \ln(c))) \left[\frac{1}{\alpha |\mathcal{N}_i|} \right]^{1+m_{ji}} (1 + o(1)) \end{aligned}$$

$$\begin{aligned} \text{PFI}(\tau^{\text{HALL}}) &\leq \max_{1 \leq i \leq N} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \text{P}_1^{(i)} \left\{ \tau^{\text{HALL}, (\mathcal{N}_j)} \leq \tau^{\text{HALL}, (\mathcal{N}_i)} \right\} \\ &\leq \frac{\exp(-(mc - (1 + \bar{m}) \ln(c)))}{\alpha^*} (1 + o(1)). \end{aligned}$$

where $\alpha^* = \min_{1 \leq i \leq N, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} (\alpha \cdot |\mathcal{N}_i|)^{1+m_{ji}}$. For any n there exists $c_0(n)$ such that for all $c > c_0(n)$, $c < e^{c/n}$. Hence, for sufficiently large c

$$\begin{aligned} \text{PFI}(\tau^{\text{HALL}}) &\leq \max_{1 \leq i \leq N} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}_i} \text{P}_1^{(i)} \left\{ \tau^{\text{HALL}, (\mathcal{N}_j)} \leq \tau^{\text{HALL}, (\mathcal{N}_i)} \right\} \\ &\leq \frac{\exp(-(m - \frac{1+\bar{m}}{n})c)}{\alpha^*} (1 + o(1)) = \frac{\exp(-b_{\text{HALL}} \cdot c)}{B_{\text{HALL}}} (1 + o(1)), \end{aligned}$$

where $b_{\text{HALL}} = m - ((1 + \bar{m})/n)$ and $B_{\text{HALL}} = \alpha^*$.

PFI – PATH–LOSS SENSING MODEL

Lemma 2 For $s \in \mathcal{N}_j \setminus \mathcal{M}_e$ and for $t \geq T$, (with the pre-change pdf $f_0 \sim \mathcal{N}(0, \sigma^2)$ and the post-change pdf $f_1 \sim \mathcal{N}(h_e \rho(r_s), \sigma^2)$)

$$\mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c \right\} \leq \exp \left(-\frac{\underline{\omega}_0}{2} c \right) \cdot \frac{\exp \left(-\frac{\alpha \underline{\omega}_0^2}{4} \right)}{1 - \exp \left(-\frac{\alpha \underline{\omega}_0^2}{4} \right)},$$

where we recall that the parameter $\underline{\omega}_0$ defines the influence range, and $\alpha = KL(f_1, f_0)$.

Proof: For $s \in \mathcal{N}_j \setminus \mathcal{M}_e$ and for $t \geq T$,

$$\begin{aligned} & \mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c \right\} \\ &= \mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ \max_{1 \leq n \leq t} \sum_{k=1}^n \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) \geq c \right\} \\ &\leq \sum_{n=1}^{\infty} \mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ \sum_{k=1}^n \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) \geq c \right\} \\ &= \sum_{n=1}^{T-1} \mathbf{P}_{\infty} \left\{ \sum_{k=1}^n \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) \geq c \right\} + \sum_{n=T}^{\infty} \mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ \sum_{k=1}^n \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) \geq c \right\} \\ &= \sum_{n=1}^{T-1} \mathbf{P}_{\infty} \left\{ \sum_{k=1}^n \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) \geq c \right\} + \sum_{n=T}^{\infty} \mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ \sum_{k=1}^{T-1} \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) + \sum_{k=T}^n \ln \left(\frac{f_1(X_k^{(s)}; r_s)}{f_0(X_k^{(s)})} \right) \geq c \right\} \\ &= \sum_{n=1}^{T-1} \mathbf{P}_{\infty} \left\{ \sum_{k=1}^n X_k^{(s)} \geq \frac{\sigma^2}{h_e \rho(r_s)} (c + n\alpha) \right\} + \sum_{n=T}^{\infty} \mathbf{P}_{\infty} \left\{ \sum_{k=1}^n X_k^{(s)} \geq \frac{\sigma^2}{h_e \rho(r_s)} c + n h_e \left(\frac{\rho(r_s)}{2} - \rho(d_{e,s}) \right) + (T - n) \right\} \\ &\leq \sum_{n=1}^{T-1} \mathbf{P}_{\infty} \left\{ \sum_{k=1}^n X_k^{(s)} \geq \frac{\sigma^2}{h_e \rho(r_s)} (c + n\alpha) \right\} + \sum_{n=T}^{\infty} \mathbf{P}_{\infty} \left\{ \sum_{k=1}^n X_k^{(s)} \geq n \cdot h_e \frac{\rho(r_s)}{2} \underline{\omega}_0 + c \cdot \frac{\sigma^2}{h_e \rho(r_s)} \right\} \\ &\leq \sum_{n=1}^{\infty} \mathbf{P}_{\infty} \left\{ \exp \left(\theta \sum_{k=1}^n X_k^{(s)} \right) \geq \exp \left(\frac{\theta \sigma^2}{h_e \rho(r_s)} (c + n\alpha \underline{\omega}_0) \right) \right\} \quad \text{for any } \theta > 0. \end{aligned}$$

$$\begin{aligned} \text{Hence, } \mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c \right\} &\leq \sum_{n=1}^{\infty} \exp \left(-\frac{\theta \sigma^2}{h_e \rho(r_s)} (c + n\alpha \underline{\omega}_0) \right) \left(\mathbb{E}_{\infty} \left[e^{\theta X_1^{(s)}} \right] \right)^n \\ &= \sum_{n=1}^{\infty} \exp \left(-\frac{\theta \sigma^2}{h_e \rho(r_s)} (c + n\alpha \underline{\omega}_0) + \frac{n \sigma^2 \theta^2}{2} \right) \end{aligned}$$

Since the above inequality holds for any $\theta > 0$, we have

$$\mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c \right\} \leq \sum_{n=1}^{\infty} \min_{\theta > 0} \exp \left(-\frac{\theta \sigma^2}{h_e \rho(r_s)} (c + n\alpha \underline{\omega}_0) + \frac{n \sigma^2 \theta^2}{2} \right)$$

The minimising θ is $\frac{c + n\alpha \underline{\omega}_0}{n h_e \rho(r_s)}$. Therefore, for $\theta = \frac{c + n\alpha \underline{\omega}_0}{n h_e \rho(r_s)}$,

$$\mathbf{P}_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c \right\} \leq \sum_{n=1}^{\infty} \exp \left(\frac{-(c + n\alpha \underline{\omega}_0)^2}{4\alpha n} \right).$$

$$\text{Note that } -\frac{(c + \alpha \underline{\omega}_0 n)^2}{4\alpha n} + \frac{(c + \alpha \underline{\omega}_0 (n-1))^2}{4\alpha (n-1)} = -\frac{\alpha \underline{\omega}_0^2}{4} + \frac{c^2}{4\alpha (n-1)n}$$

Therefore, by iteratively computing the exponent, we have

$$\begin{aligned}
\exp\left(-\frac{(c + \alpha\omega_0 n)^2}{4\alpha n}\right) &= \exp\left(-\frac{(c + \alpha\omega_0)^2}{4\alpha}\right) \cdot \exp\left(-\frac{\alpha\omega_0^2}{4}(n-1)\right) \exp\left(\frac{c^2}{4\alpha}\left(1 - \frac{1}{n}\right)\right) \\
&\leq \exp\left(-\frac{(c + \alpha\omega_0)^2}{4\alpha}\right) \cdot \exp\left(-\frac{\alpha\omega_0^2}{4}(n-1)\right) \exp\left(\frac{c^2}{4\alpha}\right) \\
\text{or } \sum_{n=1}^{\infty} \exp\left(-\frac{(c + \alpha\omega_0 n)^2}{4\alpha n}\right) &\leq \exp\left(-\frac{\omega_0}{2}c\right) \cdot \frac{\exp\left(-\frac{\alpha\omega_0^2}{4}\right)}{1 - \exp\left(-\frac{\alpha\omega_0^2}{4}\right)} \\
&=: \beta
\end{aligned}$$

D. PFI(τ^{ALL}) – Path Loss Sensing Model

$$\begin{aligned}
P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{ALL},(\mathcal{N}_j)} = t \mid \tau^{\text{ALL},(\mathcal{N}_i)} = k \right\} &\leq P_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c, \forall s \in \mathcal{N}_j \mid \tau^{\text{ALL},(\mathcal{N}_i)} = k \right\} \\
&\leq P_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c, \forall s \in \mathcal{N}_j \setminus \mathcal{N}(\ell_e) \mid \tau^{\text{ALL},(\mathcal{N}_i)} = k \right\} \\
&= \prod_{s \in \mathcal{N}_j \setminus \mathcal{N}(\ell_e)} P_1^{(\mathbf{d}(\ell_e))} \left\{ C_t^{(s)} \geq c \right\} \\
&\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \quad (\text{from Lemma 2})
\end{aligned}$$

$$\begin{aligned}
\text{Therefore, } P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{ALL},(\mathcal{N}_j)} \leq \tau^{\text{ALL},(\mathcal{N}_i)} \right\} &\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} E_1^{(\mathbf{d}(\ell_e))} \left[\tau^{\text{ALL},(\mathcal{N}_i)} \right] \\
&\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \frac{c}{\alpha |\mathcal{N}_i|} (1 + o(1))
\end{aligned}$$

Let $m = \min_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} |\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|$ and $\underline{n} = \min\{|\mathcal{N}_i| : i = 1, 2, \dots, N\}$. Define $K = \max_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} \left[\frac{\exp\left(-\frac{\alpha\omega_0^2}{4}\right)}{1 - \exp\left(-\frac{\alpha\omega_0^2}{4}\right)} \right]^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}$. Therefore,

$$\begin{aligned}
\text{PFI}(\tau^{\text{ALL}}) &\leq \max_{1 \leq i \leq N} \sup_{\ell_e \in \mathcal{A}_i} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{ALL},(\mathcal{N}_j)} \leq \tau^{\text{ALL},(\mathcal{N}_i)} \right\} \\
&\leq \frac{K \exp\left(-\left(\frac{m\omega_0}{2}c - \ln(c)\right)\right)}{\alpha \underline{n}} (1 + o(1)).
\end{aligned}$$

For any n there exists $c_0(n)$ such that for all $c > c_0(n)$, $c < e^{c/n}$. Hence, for sufficiently large c

$$\text{PFI}(\tau^{\text{ALL}}) \leq \frac{K \exp\left(-\left(\frac{m\omega_0}{2} - \frac{1}{n}\right)c\right)}{\alpha \underline{n}} (1 + o(1)) = \frac{\exp(-b_{\text{ALL},d} \cdot c)}{B_{\text{ALL},d}} (1 + o(1))$$

where $b_{\text{ALL},d} = (m\omega_0/2) - (1/n)$ and $B_{\text{ALL},d} = \alpha \underline{n}/K$.

E. PFI(τ^{MAX}) – Path Loss Sensing Model

$$\begin{aligned}
P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{MAX},(\mathcal{N}_j)} = t \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} &\leq P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{(s)} \leq t, \forall s \in \mathcal{N}_j \setminus \mathcal{N}(\ell_e) \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} \\
&= \prod_{s \in \mathcal{N}_j \setminus \mathcal{N}(\ell_e)} P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{(s)} \leq t \mid \tau^{\text{MAX},(\mathcal{N}_i)} = k \right\} \\
&\leq \prod_{s \in \mathcal{N}_j \setminus \mathcal{N}(\ell_e)} \sum_{n=1}^t P_1^{(\mathbf{d}(\ell_e))} \left\{ C_n^{(s)} \geq c \right\} \\
&\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \cdot t^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \quad (\text{from Lemma 2}) \\
P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{MAX},(\mathcal{N}_j)} \leq \tau^{\text{MAX},(\mathcal{N}_i)} \right\} &\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \cdot E_1^{(\mathbf{d}(\ell_e))} \left[(\tau^{\text{MAX},(\mathcal{N}_i)})^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \right] \\
&\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \cdot \frac{c^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}}{\alpha^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}} (1 + o(1))
\end{aligned}$$

Let $m = \min_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} |\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|$, $\bar{m} = \max_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{M}_e} |\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|$, and define $K = \max_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} \left[\frac{\exp\left(-\frac{\alpha \omega_0^2}{4}\right)}{1 - \exp\left(-\frac{\alpha \omega_0^2}{4}\right)} \right]^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}$. Therefore,

$$\begin{aligned}
\text{PFI}(\tau^{\text{MAX}}) &\leq \max_{1 \leq i \leq N} \sup_{\ell_e \in \mathcal{A}_i} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{MAX},(\mathcal{N}_j)} \leq \tau^{\text{MAX},(\mathcal{N}_i)} \right\} \\
&\leq \frac{K}{\alpha^*} \exp \left(- \left(\frac{m \omega_0}{2} c - (1 + \bar{m}) \ln(c) \right) \right) (1 + o(1)).
\end{aligned}$$

where $\alpha^* = \min_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} \alpha^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}$. For any n there exists $c_0(n)$ such that for all $c > c_0(n)$, $c < e^{c/n}$. Hence, for sufficiently large c

$$\text{PFI}(\tau^{\text{MAX}}) \leq \frac{K}{\alpha^*} \exp \left(- \left(\frac{m \omega_0}{2} - \frac{1 + \bar{m}}{n} \right) c \right) (1 + o(1)) = \frac{\exp(-b_{\text{MAX},d} \cdot c)}{B_{\text{MAX},d}} (1 + o(1)),$$

where $b_{\text{MAX},d} = \left(\frac{m \omega_0}{2} \right) - \left(\frac{1 + \bar{m}}{n} \right)$ and $B_{\text{MAX},d} = \frac{\alpha^*}{K}$.

F. PFI(τ^{HALL}) – Path Loss Sensing Model

$$P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{HALL},(\mathcal{N}_j)} = t \mid \tau^{\text{HALL},(\mathcal{N}_i)} = k \right\} \leq P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{(s)} \leq t, \forall s \in \mathcal{N}_j \setminus \mathcal{N}(\ell_e) \mid \tau^{\text{HALL},(\mathcal{N}_i)} = k \right\}$$

which has the same form as that of MAX. Hence, from the analysis of MAX, it follows that

$$\begin{aligned}
P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{HALL},(\mathcal{N}_j)} \leq \tau^{\text{HALL},(\mathcal{N}_i)} \right\} &\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} E_1^{(\mathbf{d}(\ell_e))} \left[(\tau^{\text{HALL},(\mathcal{N}_i)})^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \right] \\
&\leq \beta^{|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|} \frac{c^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}}{(\alpha^{|\mathcal{N}_i|})^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}} (1 + o(1))
\end{aligned}$$

$$\begin{aligned}
\text{Therefore, } \text{PFI}(\tau^{\text{HALL}}) &\leq \max_{1 \leq i \leq N} \sup_{\ell_e \in \mathcal{A}_i} \max_{1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} P_1^{(\mathbf{d}(\ell_e))} \left\{ \tau^{\text{HALL},(\mathcal{N}_j)} \leq \tau^{\text{HALL},(\mathcal{N}_i)} \right\} \\
&\leq \frac{K}{\alpha^*} \exp \left(- \left(\frac{m \omega_0}{2} c - (1 + \bar{m}) \ln(c) \right) \right) (1 + o(1)).
\end{aligned}$$

Therefore for large c , $\text{PFI} \leq \frac{K}{\alpha^*} \exp \left(- \left(\frac{m\omega_0}{2} - \frac{1 + \bar{m}}{n} \right) c \right) (1 + o(1)) = \frac{\exp(-b_{\text{HALL},d} \cdot c)}{B_{\text{HALL},d}} (1 + o(1))$,

where $\alpha^* = \min_{1 \leq i \leq N, \ell_e \in \mathcal{A}_i, 1 \leq j \leq N, \mathcal{N}_j \not\subseteq \mathcal{N}(\ell_e)} (\alpha \cdot |\mathcal{N}_i|)^{1+|\mathcal{N}_j \setminus \mathcal{N}(\ell_e)|}$, $b_{\text{HALL},d} = (m\omega_0/2) - (1 + \bar{m})/n$, and $B_{\text{HALL},d} = \alpha^*/K$.

APPENDIX C

SADD FOR THE BOOLEAN AND THE PATH LOSS MODELS

Fix $i, 1 \leq i \leq N$. For each change time $T \geq 1$, define $\mathcal{F}_T = \sigma(X_k^{(s)}, s \in \mathcal{N}, 1 \leq k \leq T)$, and for $\ell_e \in \mathcal{A}_i$, $\mathcal{F}_T^{(i)} = \sigma(X_k^{(s)}, s \in \mathcal{N}_i, 1 \leq k \leq T)$. From [9] (Theorem 3, Eqn. (24)),

$$\text{ess sup } \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{(T-1)}^{(i)} \right) \leq \frac{c}{I} (1 + o(1)), \text{ as } c \rightarrow \infty, \quad (28)$$

Define $\mathcal{F}_{\{\tau^{\text{rule}, (\mathcal{N}_i)} \geq T\}}$ as the σ -field generated by the event $\{\tau^{\text{rule}, (\mathcal{N}_i)} \geq T\}$, and similarly define the σ -field $\mathcal{F}_{\{\tau^{\text{rule}} \geq T\}}$. Evidently $\mathcal{F}_{\{\tau^{\text{rule}, (i)} \geq T\}} \subset \mathcal{F}_{(T-1)}^{(i)}$ and $\mathcal{F}_{\{\tau^{\text{rule}} \geq T\}} \subset \mathcal{F}_{(T-1)}$. By iterated conditional expectation,

$$\mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{\{\tau^{\text{rule}} \geq T\}} \right) \leq \text{ess sup } \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{(T-1)} \right) \quad (29)$$

We can further assert that

$$\mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{(T-1)} \right) \stackrel{\text{a.s.}}{=} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{(T-1)}^{(i)} \right)$$

Using this observation with Eqn. 29 and Eqn. 28, we can write, as $c \rightarrow \infty$,

$$\mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{\{\tau^{\text{rule}} \geq T\}} \right) \leq \frac{c}{I} (1 + o(1)) \quad (30)$$

Finally, $\mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \tau^{\text{rule}} \geq T \right) I_{\{\tau^{\text{rule}} \geq T\}} \stackrel{\text{a.s.}}{=} \mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \mathcal{F}_{\{\tau^{\text{rule}} \geq T\}} \right) I_{\{\tau^{\text{rule}} \geq T\}}$.

We conclude, from 30, that, as $c \rightarrow \infty$, $\mathbb{E}_T^{(\mathbf{d}(\ell_e))} \left((\tau^{\text{rule}, (\mathcal{N}_i)} - T)^+ | \tau^{\text{rule}} \geq T \right) \leq \frac{c}{I} (1 + o(1))$.

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